What are Wingsuit Equations?

$$\frac{\dot{\vec{V}}}{g} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - V \begin{bmatrix} K_d & -K_l \\ K_l & K_d \end{bmatrix} \vec{V}$$

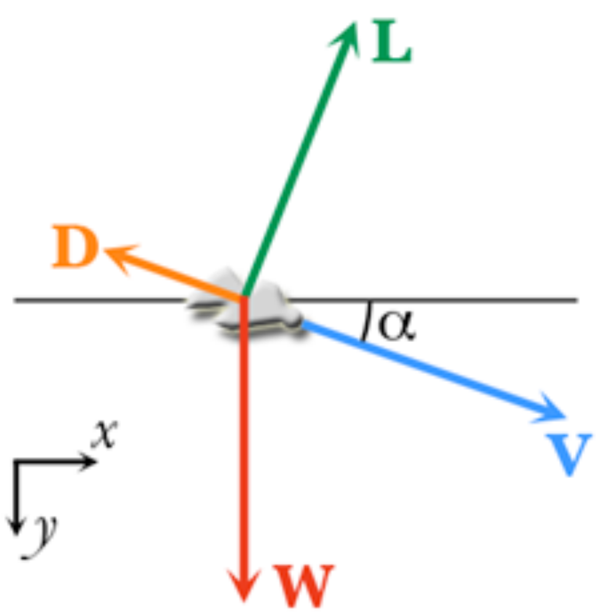
Wingsuit Equations are differential equations of motion governing wingsuit flight dynamics in two dimensions. Simple and beautiful, they allow for high precision simulation of wingsuit flight by knowing sustained velocity alone.

When we think about modeling of wingsuit flight, the task at hand seems to be impossible to solve. To calculate trajectory, one would need to precisely know forces acting on the body at any moment in time so that acceleration can be calculated. Given initial velocity, one can then numerically integrate acceleration to obtain velocity at any moment, and then further integrate the velocity to obtain complete trajectory. But how can we know the forces?

The aerodynamic forces acting on a flying body – lift and drag – are proportional to density of air, surface area, lift and drag coefficients, respectively, and the square of airspeed. We do not know exactly what the surface area and lift and drag coefficients are, and the latter depend on the angle of attack and body position. It seems, we cannot calculate the aerodynamic forces and thus cannot model the flight.

But with a little help of *pure flying magic*… we can!

Let us consider the force diagram of a flyer. We have 3 forces: weight $W=mg$, lift $L=½C_l·\rho·S·V^2$ which is perpendicular to current velocity V, and drag $D=½C_d·\rho·S·V^2$ which is opposite to velocity. Here C_l and C_d are nondimensional coefficients of lift and drag, respectively, ρ is the density of air, and S is the effective area of the wingsuit.



If the current glide angle to horizon is α, then the horizontal components of lift and drag are $L\sin\alpha$ and $-D\cos\alpha$, vertical components are $-L\cos\alpha$ and $-D\sin\alpha$. Therefore, we have these two Newton's equations of motion:

$$F_x = ma_x = L\sin\alpha - D\cos\alpha$$
$$F_y = ma_y = mg - L\cos\alpha - D\sin\alpha$$

where $a_x = dV_x/dt$, $a_y = dV_y/dt$ are horizontal and vertical accelerations, V_x and V_y are horizontal and vertical components of total speed

$$V = \sqrt{V_x^2 + V_y^2}$$

Since $\sin\alpha = V_y/V$, $\cos\alpha = V_x/V$, the equations can be refactored to

$$\frac{dV_x}{dt} = \frac{1}{2}\frac{\rho S}{m} V\left(C_l V_y - C_d V_x\right)$$
$$\frac{dV_y}{dt} = g - \frac{1}{2}\frac{\rho S}{m} V\left(C_l V_x + C_d V_y\right)$$

These equations seem to support the doubt that they can be successfully solved as we do not know C_l, C_d, and S. One would need to do extensive windtunnel experiments to find C_l and C_d for various angles of attack.

Here comes the magic. Let us introduce “magic” lift and drag coefficients defined as

$$K_l = \frac{1}{2}\frac{\rho S}{mg}C_l$$

$$K_d = \frac{1}{2}\frac{\rho S}{mg}C_d$$

The equations of motion now become

$$\frac{dV_x}{dt} = gV\left(K_l V_y - K_d V_x\right)$$

$$\frac{dV_y}{dt} = g - gV\left(K_l V_x + K_d V_y\right)$$

But we still do not know K_l and K_d… or do we?

Suppose that with the current body position and angle of attack the pilot's sustained horizontal and vertical speeds are V_{xs} and V_{ys}. Since for sustained flight acceleration is zero by definition, the equations read

$$gV_s(K_l V_{ys} - K_d V_{xs}) = 0$$
$$g - gV_s(K_l V_{xs} + K_d V_{ys}) = 0$$

where V_s is the total sustained speed. These two equations with two unknowns are easily solved for K_l and K_d:

$$K_l = \frac{V_{xs}}{V_s^3}$$
$$K_d = \frac{V_{ys}}{V_s^3}$$

Note that the ratio of these coefficients is equal to lift-to-drag ratio:

$$\frac{K_l}{K_d} = \frac{V_{xs}}{V_{ys}} = \frac{L}{D}$$

Now the unknown aerodynamic parameters (wingloading *mg/S* and aerodynamic coefficients C_l, C_d) are "hidden" inside coefficients K_l and K_d, which can be easily calculated from sustained horizontal and vertical speeds for a given flight mode, enabling us to solve the equations of motion. See, sometimes you just need to shove the problems under the carpet and all of a sudden it becomes a magic carpet that takes you for an amazing ride. It's magic… pure flying magic!

The system of differential equations we just derived is called Wingsuit Equations:

$$\frac{dV_x}{dt} = gV\left(K_l V_y - K_d V_x\right)$$

$$\frac{dV_y}{dt} = g - gV\left(K_l V_x + K_d V_y\right)$$

They can also be written in elegant matrix form:

$$\frac{\dot{\vec{V}}}{g} = \begin{bmatrix}0\\1\end{bmatrix} - V\begin{bmatrix}K_d & -K_l\\ K_l & K_d\end{bmatrix}\vec{V}$$

The simplest case is when K_l and K_d are constant throughout the flight (meaning that the pilot maintains the same body position and angle of attack at all times). Suppose, you measured your sustained speeds for your best L/D on a skydive using GPS in no wind conditions and they are V_{xs}=90mph, V_{ys}=36mph (L/D=2.5). If you make a BASE jump while maintaining exactly the same body position and varying your pitch angle as to keep the same angle of attack at all times, then we have Wingsuit Equations with constant known coefficients K_l and K_d calculated from sustained speeds as shown above and they can easily be solved numerically. By knowing just two numbers measured on a skydive, we are able to calculate position and velocity at each moment of a BASE jump!

If angle of attack and body position are not constant throughout the flight, we need to know various flight modes (combinations of V_{xs} and V_{ys}, or, equivalently, K_l and K_d) at different angles of attack. Flight modes can be measured by inducing sustained flight in each mode on a skydive or by analyzing transitional (i.e. non-sustained) flight data on a BASE jump. When integrating WSE, a flight mode corresponding to the current angle of attack is used to calculate the state of flight in the next moment in time, and so on.

How practical is the assumption of a constant flight mode on a BASE jump? In the first 2-3 seconds while you are transitioning from your exit into flight, the aerodynamic forces are small enough that the substantial variation in the angle of attack and correspondingly, lift and drag coefficients, does not have any significant effect on overall flight. Best flights feel smooth and efficient from exit to deployment – you quickly and smoothly transition into steep dive with low angle of attack, “freeze” your body position and naturally change your pitch angle into the flight as to keep the angle of attack close to your most efficient maxed out mode. In this case, the assumption of constant flight mode is good enough to be practical. If you compare trajectories calculated using Wingsuit Equations, with your GPS data, you will see a strikingly good agreement between theory and practice – at least for flights which feel smooth and efficient.

Wingsuit Equations were called as such because they were first derived while specifically researching wingsuit flight dynamics. While they are equally applicable to any form of nonpowered 2-dimensional gliding flight, their practical usefulness for wingsuit flying (especially for WS BASE) makes them of special interest for wingsuit flyers, hence the name.

Wingsuit Equations were first published on dropzone.com forum on December 7, 2006:
http://www.dropzone.com/cgi-bin/forum/gforum.cgi?post=2563135